

Collective modes in the color flavor locked phase

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Abstract. We study the low energy effective action for some collective modes of the color flavor locked phase of QCD. This phase of matter has long been known to be a superfluid because by picking a phase its order parameter breaks the quark-number $U(1)_B$ symmetry spontaneously. We consider the modes describing fluctuations in the magnitude of the condensate, namely the Higgs mode, and in the phase of the condensate, namely the Nambu-Goldstone (or Anderson-Bogoliubov) mode associated with the breaking of $U(1)_B$. By employing as microscopic theory the Nambu-Jona Lasinio model, we reproduce known results for the Lagrangian of the Nambu-Goldstone field to the leading order in the chemical potential and extend such results evaluating corrections due to the gap parameter. Moreover, we determine the interaction terms between the Higgs and the Nambu-Goldstone field. This study paves the way for a more reliable study of various dissipative processes in rotating compact stars with a quark matter core in the color flavor locked phase.

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1. Introduction

At extremely high densities, Quantum chromodynamics (QCD) predicts that the individual nucleons that form standard hadronic matter should melt and their quark matter content should be liberated [1]. At the low temperatures expected in sufficiently old compact stars, quark matter is likely to be in one of the possible color superconducting phases, whose critical temperatures are generically of order tens of MeV [2, 3]. Since compact star temperatures are well below these critical temperatures, for many purposes the quark matter that may be found within compact stars can be approximated as having zero temperature, as we shall assume throughout. At asymptotic densities, where the up, down and strange quarks can be treated on an equal footing and effects due to the strange quark mass can be neglected, quark matter is in the color flavor locked (CFL) phase [4, 2]. The CFL condensate is antisymmetric in color and flavor indices and involves pairing between up, down and strange quarks. The order parameter breaks the quark-number $U(1)_B$ symmetry spontaneously, and the corresponding Nambu-Goldstone (NG) boson (Anderson-Bogoliubov mode) determines the superfluid properties of CFL quark matter. The gapless excitation corresponds to a phase oscillation about the mean field value of the gap parameter. The fluctuation in magnitude of the condensate is associated with a massive mode, which we shall refer to as the Higgs mode. Both of these fluctuations are excitations of several fermions and therefore describe collective modes of the system.

The low energy properties of the system are completely determined by these collective modes [5], and their study is mandatory to understand dynamical properties that take place in compact stars with a CFL core. The actual superfluid property of the system is due to the presence of the gapless excitations, which satisfy the Landau's criterion for superfluidity [6]. Moreover, in rotating superfluids the interactions between NG bosons and vortices lead to the appearance of the so-called mutual friction force between the normal and the superfluid components of the system. The NG bosons are also responsible of many other properties of cold superfluid matter, in particular they contribute to the thermal conductivity and to the shear and bulk viscosities [6]. A study of the shear viscosity and of the standard bulk viscosity coefficient due to NG bosons has been done in Refs. [7, 8]. A more detailed study of the bulk viscosity coefficients of the CFL phase has been done in [9]. In the CFL phase also the global $SU(3)_R \times SU(3)_L$ symmetry is spontaneously broken and the corresponding pseudo-NG bosons can contribute to the transport properties of the CFL phase. The contribution of kaons to the bulk and shear viscosity has been studied in Refs. [10, 11]. The contribution of NG bosons to the thermal conductivity and cooling of compact stars were studied in Refs. [12, 13, 14, 15]. Other low energy degrees of freedom are the plasmons, which have been studied in Refs. [16, 17].

Although the CFL phase is characterized by many low energy degrees of freedom, in the present paper we shall focus on the Nambu-Goldstone field, ϕ , and on the Higgs field, ρ , because we aim to build the effective Lagrangian for the description of NG

boson-vortex interaction. Vortices are described by spatial variation of the condensate and therefore are determined by the space variation of the Higgs field. In principle one could study interaction of vortices with other low energy degrees of freedom, but the NG boson associated to the breaking of $U(1)_B$ is the only massless mode, whereas pseudo-NG bosons associated to the breaking of chiral symmetry have a mass of the order of few keV [18], and plasmons associated to the breaking of $SU(3)_c$ symmetry have even larger masses [16, 17]. Therefore all of these modes are thermally suppressed in cold compact stars.

We derive the effective Lagrangian of the Nambu-Goldstone field, and of the Higgs field using as microscopic theory the Nambu-Jona Lasinio model [19, 20] with a local four-Fermi interaction with the quantum numbers of one gluon exchange. This model mimics some aspects of QCD at large densities [21, 22]. We derive the interaction terms of the NG bosons up to terms of the type $(\partial\phi)^4$. The leading contributions to these interaction terms were obtained by Son in Ref. [23] using symmetry arguments and the expression of the pressure of the CFL phase. To our knowledge the results of Ref. [23] so far have not been obtained starting from a microscopic theory. Only the free Lagrangian was obtained in [18]. We extend the results of [23] including the next to leading corrections proportional to the gap parameter. In order to do this we integrate out the fermionic degrees of freedom employing the High Density Effective Theory (HDET) [3]. Moreover we determine the kinetic Lagrangian for the Higgs field and the interaction terms between the Higgs and the NG bosons.

A similar analysis to the one presented here was done for non-relativistic systems at unitarity in [24] and in [25, 26]. As we shall discuss in Section 4, the main difference with the non-relativistic systems at unitarity is that in the CFL phase integrating out the Higgs field does not lead to a change of the speed of sound.

This paper is organized as follows. In Section 2 we present the Nambu-Jona Lasinio model and we determine the expression of the effective action of the system in the HDET approximation. In Section 3 we derive the effective Lagrangian for the NG boson, neglecting the oscillations in the magnitude of the condensate. In Section 4 we derive the LO interaction terms between the Higgs mode and the NG bosons and the kinetic Lagrangian for the Higgs field. We also compare our results with the non-relativistic results of [24]. We draw our conclusions in Section 5. Some interaction vertices as well as the calculation of some integrals are reported in the Appendix.

2. The model

We consider a Nambu-Jona Lasinio (NJL) model of quark matter with a local four-Fermi interaction as a model of QCD at large quark chemical potential μ . We assume that $\mu \gg m_s$ and therefore we neglect u , d and s quark masses. In the absence of interactions the Lagrangian density describing the system of up, down and strange quarks is given by

$$\mathcal{L}_0 = \bar{\psi}_{i\alpha}(i\not{D} + \mu\gamma_0)\psi_{i\alpha}, \quad (1)$$

where $i, j = 1, 2, 3$ are flavor indices, $\alpha, \beta = 1, 2, 3$ are color indices and the Dirac indices have been suppressed. In QCD one can show that at large densities the gluon exchange leads to the formation of a quark-quark condensate [2]. At asymptotic densities the favored phase is the color flavor locked phase [4] which is characterized by the condensate

$$\langle \psi_{i\alpha}^t C \psi_{j\beta} \rangle \sim \Delta \sum_{I=1,2,3} \epsilon_{\alpha\beta I} \epsilon^{ijI}, \quad (2)$$

which locks together color and flavor rotations. Here $C = i\gamma^2\gamma^0$ is the charge conjugation matrix and $\epsilon_{\alpha\beta I}$ and ϵ^{ijI} are Levi-Civita tensors. This condensate breaks several symmetries of QCD, see [4] for a detailed analysis. For our purposes it is sufficient to note that the CFL condensate breaks the $U(1)_B$ symmetry and therefore leads to the appearance of a gapless Nambu-Goldstone boson.

In the NJL model the interaction among quarks mediated by gluons is replaced by a four-Fermi interaction of the BCS type. We shall consider an interaction with the same quantum numbers of one gluon exchange

$$\mathcal{L}_1 = -\frac{3}{16}g \bar{\psi} \gamma_\mu \lambda^A \psi \bar{\psi} \gamma^\mu \lambda^A \psi, \quad (3)$$

where $g > 0$ is the coupling constant and λ^A with $A = 1, \dots, 8$ are the Gell-Mann matrices. This interaction at large chemical potentials leads to the formation of the CFL condensate. Introducing the new basis for the quark fields

$$\psi_{i\alpha} = \frac{1}{\sqrt{2}} \sum_{A=1}^9 \lambda_{i\alpha}^A \psi_A, \quad (4)$$

where $\lambda^9 = \sqrt{2/3} \times I$, we have that

$$\langle \psi_A^t C \psi_B \rangle \sim \Delta_{AB}, \quad (5)$$

where $\Delta_{AB} = \Delta_A \delta_{AB}$, where $\Delta_1 = \dots = \Delta_8 = \Delta$ and $\Delta_9 = -2\Delta$. In this basis the four-Fermi interaction is given by

$$\mathcal{L}_1 = -\frac{g}{4} V_{ABCD} \epsilon_{ab} \epsilon_{\dot{c}\dot{d}} \psi_a^A \psi_b^B \psi_{\dot{c}}^{C\dagger} \psi_{\dot{d}}^{D\dagger}, \quad (6)$$

where $V_{ABCD} = \text{Tr} \sum_{E=1}^8 (\lambda_A \lambda_E \lambda_B \lambda_C \lambda_E \lambda_D)$ and $a(\dot{a}) = 1, 2$ are the Weyl indices for $L(R)$ components [27].

In order to study the fluctuation of the condensate we introduce the Hubbard-Stratonovich fields $\Delta_{AB}(x)$ and $\Delta_{AB}^*(x)$ which allow to write the partition function (normalized at the free case for $g = 0$) as

$$\frac{\mathcal{Z}}{\mathcal{Z}_0} = \frac{1}{\mathcal{Z}_0} \int [d\psi, d\psi^\dagger] [d\Delta, d\Delta^*] \exp \left\{ i \int d^4x \left[-\frac{\Delta_{AB} W_{ABCD} \Delta_{CD}^*}{g} + \mathcal{L}_\Delta \right] \right\}, \quad (7)$$

where

$$W_{ABCD} V_{CDEF} = \delta_{AE} \delta_{BF} \quad \text{and} \quad V_{ABCD} W_{CDEF} = \delta_{AE} \delta_{BF}, \quad (8)$$

and the semi-bosonized Lagrangian is given by

$$\mathcal{L}_\Delta = \bar{\psi}_A (i\not{\partial} + \mu\gamma_0) \psi_A - \frac{1}{2} \Delta_{AB} (\psi_A^\dagger C \psi_B^*) + \frac{1}{2} \Delta_{AB}^* (\psi_A^t C \psi_B). \quad (9)$$

The fluctuations of the condensate $\Delta(x)$ around the mean field value Δ^{MF} can be described by two real fields; one field describes the variation of $|\Delta(x)|$ while the other field is associated to a local phase change. Therefore we write

$$\Delta_{AB}(x) = [\Delta_{AB}^{MF} + \rho_{AB}(x)]e^{2i\phi(x)}, \quad (10)$$

where $\rho_{AB}(x) = \rho(x) \text{diag}(1, \dots, 1, -2)_{AB}$ and $\phi(x)$ are the real fields. Hereafter we shall suppress the indices A, B and we shall indicate with Δ the mean field value of the gap in order to simplify the notation. We find convenient to redefine the fermionic fields as

$$\psi \rightarrow \psi e^{i\phi(x)}, \quad (11)$$

and in this way the semi-bosonized Lagrangian is given by

$$\begin{aligned} \mathcal{L}_\Delta = & \bar{\psi} \left(i\gamma^\mu \partial_\mu + \gamma^0 \mu - \gamma^0 \partial_0 \phi - \gamma^i \partial_i \phi \right) \psi \\ & - \frac{1}{2} \psi^\dagger C(\Delta + \rho) \psi^* + \frac{1}{2} \psi^t C(\Delta + \rho) \psi. \end{aligned} \quad (12)$$

Writing the Lagrangian in this form it is possible to define an “effective chemical potential”

$$\tilde{\mu} = \mu - \partial_0 \phi. \quad (13)$$

In other words $\partial_0 \phi$ describes long-wavelength fluctuations of the chemical potential on the top of its constant value μ . We want now to clarify one point regarding the terminology used for the field ϕ . Since $\partial_0 \phi$ describes fluctuations of the chemical potential, it is not correct to call this field the *phonon*, which describes pressure oscillations. However, when one neglects the effect of the gap Δ , the Lagrangian of the phonon and of the NG boson associated with the breaking of $U(1)_B$ symmetry coincide. The reason is that for vanishing values of Δ the oscillations of pressure are proportional to the oscillations of the chemical potential.

It is convenient to define a fictitious gauge field $A^\mu = (\partial_0 \phi, \nabla \phi)$ which according with Eq. (12) is minimally coupled to the quark fields. Hereafter we shall refer to A^μ as the gauge field, although it is not related to any gauge symmetry of the system. With this substitution we rewrite the Lagrangian in Eq. (12) as

$$\mathcal{L}_\Delta = \bar{\psi} \left(i\gamma^\mu D_\mu + \mu \gamma^0 \right) \psi - \frac{1}{2} \psi^\dagger C(\Delta + \rho) \psi^* + \frac{1}{2} \psi^t C(\Delta + \rho) \psi, \quad (14)$$

where $D_\mu = \partial_\mu + iA_\mu$.

In order to simplify the calculation we employ the High Density Effective Theory (HDET), see [3]. Using standard techniques, the Lagrangian describing the kinetic terms and the interaction with the gauge field A^μ can be written as

$$\begin{aligned} \mathcal{L}_I = & \int \frac{d\mathbf{v}}{8\pi} \left[\psi_+^\dagger \left(iV \cdot D - \frac{P^{\mu\nu} D_\mu D_\nu}{2\mu + i\tilde{V} \cdot D} \right) \psi_+ \right. \\ & \left. + \psi_-^\dagger \left(i\tilde{V} \cdot D - \frac{P^{\mu\nu} D_\mu D_\nu}{2\mu + iV \cdot D} \right) \psi_- \right], \end{aligned} \quad (15)$$

where the positive energy fields with “positive” and “negative” velocities are given by

$$\psi_\pm \equiv \psi_+(\pm v), \quad (16)$$

and where

$$V^\mu = (1, \mathbf{v}), \quad \tilde{V}^\mu = (1, -\mathbf{v}), \quad P^{\mu\nu} = g^{\mu\nu} - \frac{V^\mu \tilde{V}^\nu + V^\nu \tilde{V}^\mu}{2}, \quad (17)$$

with \mathbf{v} the Fermi velocity. The non-local interactions which are present in Eq. (15) are due to the integration of the negative energy fields [3].

In order to have a compact notation it is useful to introduce the Nambu–Gorkov spinor

$$\Psi = \frac{1}{\sqrt{2}} \begin{pmatrix} \psi_+ \\ C\psi_-^* \end{pmatrix}, \quad (18)$$

which allows to write the semi-bosonized Lagrangian as

$$\mathcal{L}_\Delta = \mathcal{L}_a + \mathcal{L}_b, \quad (19)$$

where

$$\mathcal{L}_a = \int \frac{d\mathbf{v}}{4\pi} \Psi^\dagger \begin{pmatrix} iV^\mu \partial_\mu - V^\mu A_\mu & \Delta + \rho \\ \Delta + \rho & i\tilde{V}^\mu \partial_\mu + \tilde{V}^\mu A_\mu \end{pmatrix} \Psi, \quad (20)$$

is the sum of the kinetic term and of the local interaction term, while the non-local interaction term is given by

$$\mathcal{L}_b = - \int \frac{d\mathbf{v}}{4\pi} P^{\mu\nu} \Psi^\dagger \begin{pmatrix} \frac{-2\mu + iV \cdot D^*}{L} D_\mu D_\nu & \frac{\Delta}{L} D_\mu^* D_\nu \\ \frac{\Delta}{L} D_\mu D_\nu^* & \frac{2\mu + i\tilde{V} \cdot D}{L} D_\mu^* D_\nu^* \end{pmatrix} \Psi, \quad (21)$$

where $L = (2\mu + i\tilde{V} \cdot D)(-2\mu + iV \cdot D^*) - \Delta^2 - i\epsilon$. More details about the derivation of the non-local interaction will be given in [28].

The semi-bosonized Lagrangian is quadratic in the fermionic fields and therefore we can integrate them out. In this way the effective action can be written in terms of the fields ϕ and ρ and their derivatives. An analogous calculation in the non-relativistic case has been done in [25]. Before doing this we consider in more detail the interaction terms between the fermionic fields and the fictitious gauge field determined by the non-local term. In momentum space we have that

$$\mathcal{L}_b = \int \frac{d\mathbf{v}}{4\pi} \Psi^\dagger P^{\mu\nu} A_\mu A_\nu \begin{pmatrix} -2\mu + V \cdot \ell + V \cdot A & -\Delta \\ -\Delta & 2\mu + \tilde{V} \cdot \ell - \tilde{V} \cdot A \end{pmatrix} \frac{1}{L} \Psi, \quad (22)$$

where in momentum space $L = (2\mu + \tilde{V} \cdot \ell - \tilde{V} \cdot A)(-2\mu + V \cdot \ell + V \cdot A) - \Delta^2 - i\epsilon$. The “residual momentum” of the quarks, ℓ^μ , is defined as follows:

$$\ell_0 = p_0 \quad \ell_i = p_i - \mu v_i, \quad (23)$$

where p^μ is the four-momentum of the quarks.

Expanding the denominator of the expression in Eq. (22) in powers of A and considering terms up to the order A^4 , we have that

$$\begin{aligned} \mathcal{L}_b = & \int \frac{d\mathbf{v}}{4\pi} \left(\Psi^\dagger \Gamma_2^{\mu\nu} \Psi A_\mu A_\nu + \Psi^\dagger \Gamma_3^{\mu\nu\rho} \Psi A_\mu A_\nu A_\rho \right. \\ & \left. + \Psi^\dagger \Gamma_4^{\mu\nu\rho\sigma} \Psi A_\mu A_\nu A_\rho A_\sigma \right) + \mathcal{O}(A^5), \end{aligned} \quad (24)$$



Figure 1. The tree-level interaction vertices of the gauge field with the fermions in the HDET. Full lines correspond to fermionic fields; wavy lines correspond to the gauge fields. The vertices are named Γ_1 , Γ_2 , Γ_3 and Γ_4 , where the subscripts indicates the number of the external gauge fields. Their expression is reported in the Appendix A.



Figure 2. Interaction of the Higgs field (dashed line) with quarks (full line). The corresponding vertex is named Γ_ρ and is reported in Eq. (29).

where the expression of the vertices Γ_2 , Γ_3 and Γ_4 are reported in the Appendix A. We also define

$$\Gamma_1^\mu = \begin{pmatrix} -V^\mu & 0 \\ 0 & \tilde{V}^\mu \end{pmatrix}, \quad (25)$$

which is the vertex that describes the minimal coupling of quarks with the gauge field in the HDET. The various vertices are schematically depicted in Fig. 1, with the wavy lines corresponding to the gauge field and the full line corresponding to the fermionic field.

Integrating out the fermionic fields, the partition function turns out to be given by

$$\frac{\mathcal{Z}}{\mathcal{Z}_0} = \frac{\int [d\Delta, d\Delta^*] \exp \left[\frac{1}{g} \int d^4x \Delta_{AB} W_{ABCD} \Delta_{CD}^* \right] \det[S^{-1}]^{1/2}}{\det[S_0^{-1}]^{1/2}} \equiv \exp[i\mathcal{S}] \quad (26)$$

where \mathcal{S} is the action of the system and the full inverse propagator is given by

$$S^{-1} \equiv S_{MF}^{-1} + \Gamma. \quad (27)$$

The mean field inverse propagator is given by

$$S_{MF}^{-1} = \begin{pmatrix} iV^\mu \partial_\mu & -\Delta \\ -\Delta & i\tilde{V}^\mu \partial_\mu \end{pmatrix}, \quad (28)$$

while $\Gamma = \Gamma_\rho + \Gamma_1 + \Gamma_2 + \Gamma_3 + \Gamma_4$. The interaction of quarks with the ρ field is described by the vertex

$$\Gamma_\rho = \begin{pmatrix} 0 & -\rho \\ -\rho & 0 \end{pmatrix}, \quad (29)$$

while the interaction of quarks with the NG bosons is given by $\Gamma_1 = \Gamma_1^\mu A_\mu$, $\Gamma_2 = \Gamma_2^{\mu\nu} A_\mu A_\nu$, $\Gamma_3 = \Gamma_3^{\mu\nu\rho} A_\mu A_\nu A_\rho$ and $\Gamma_4 = \Gamma_4^{\mu\nu\rho\sigma} A_\mu A_\nu A_\rho A_\sigma$.

We separate the mean field action from the fluctuation writing

$$\mathcal{S} = \mathcal{S}_{MF} + \mathcal{S}_{\text{eff}}, \quad (30)$$

where \mathcal{S}_{eff} is the effective action describing the low energy properties of the system. The mean field action provides the free energy of the system

$$\Omega = \mathcal{S}_{MF} = -\frac{i}{2}\text{Tr}\ln[S_{MF}^{-1}] - \frac{i}{g}[\Delta_{AB}W_{ABCD}\Delta_{CD}^*], \quad (31)$$

and is a function of the quark gap parameter Δ . Hereafter, Tr , symbolizes the trace over the Nambu-Gorkov index, the trace over color-flavor indices, the trace over spinorial indices and the trace over a complete set of functions in space-time.

The gap parameter can be determined by the stationary condition of the mean field action

$$\left.\frac{\partial\mathcal{S}_{MF}}{\partial\Delta}\right|_{\bar{\Delta}} = 0, \quad (32)$$

which in turn allows to determine the pressure of the system by $P = -\Omega(\bar{\Delta})$. In the NJL model, the mean field value of the gap parameter turns out to depend on the coupling g and on the three-momentum cutoff, see *e.g.* [3]. Since we do not need the numerical value of the gap, we shall treat Δ as a free parameter.

The fluctuation around the mean field solution are described by the effective action

$$\begin{aligned} \mathcal{S}_{\text{eff}} = & -\frac{i}{g}\int d^4x [\rho_{AB}(x)W_{ABCD}\rho_{CD}(x) + 2\rho_{AB}(x)W_{ABCD}\Delta_{CD}] \\ & -\frac{i}{2}\text{Tr}\ln(1 + S_{MF}\Gamma), \end{aligned} \quad (33)$$

which contains the Γ expansion

$$\text{Tr}\ln(1 + S_{MF}\Gamma) = \text{Tr}\left[\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}(S_{MF}\Gamma)^n\right], \quad (34)$$

and we shall evaluate terms up to the fourth order in the gauge fields. We shall also determine the leading order terms of the ρ field Lagrangian and the leading order interaction terms of the ρ field with the NG bosons. In the expansion we shall neglect the NLO terms of the kind $(\partial\partial\phi)^n$ where n is any nonzero integer.

3. The Lagrangian for the NG boson

Neglecting oscillations in the modulus of the condensate we can determine from the Eqs. (33) and (34) the effective Lagrangian for the NG bosons. As we shall show below, for vanishing values of Δ we obtain the same results obtained in Ref. [23]

$$\mathcal{L}_{\phi} = \frac{3}{4\pi^2} [(\mu - \partial_0\phi)^2 - (\partial_i\phi)^2]^2. \quad (35)$$

This expression relies on symmetry considerations, in particular on conformal symmetry, and on the expression of the pressure in the CFL phase. We shall reproduce these results and evaluate the corrections of the order $(\Delta/\mu)^2$ which are related with the breaking of conformal symmetry. Our strategy is to first expand the Lagrangian in the A_{μ} fields, and write

$$\mathcal{L}_{\phi} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4, \quad (36)$$

where \mathcal{L}_m is the term with m gauge fields. Then we expand the various terms in Δ/μ . Notice that the term \mathcal{L}_m will be obtained by the expansion in Eq. (34) considering all the terms with $n \leq m$.

A different way of obtaining the leading order in μ Lagrangian for terms proportional to $\partial_0\phi$ is the following. At the leading order in μ — neglecting terms proportional to Δ — the free-energy density of the CFL phase in the absence of oscillations is given by

$$\Omega = \frac{3}{4\pi^2}\mu^4. \quad (37)$$

We have seen in Section 2 that $\partial_0\phi$ corresponds to a fluctuation of the chemical potential of the system. Therefore, including these fluctuations the free-energy density of the CFL phase at the leading order in μ is given by

$$\tilde{\Omega} = \frac{3}{4\pi^2}\tilde{\mu}^4, \quad (38)$$

where $\tilde{\mu}$ is defined in Eq. (13). Expanding the free energy we obtain that the Lagrangian for the $\partial_0\phi$ field is given by

$$\mathcal{L}(\partial_0\phi) = \frac{3}{4\pi^2}\mu^4 - \frac{3}{\pi^2}\mu^3\partial_0\phi + \frac{9}{2\pi^2}\mu^2(\partial_0\phi)^2 - \frac{3\mu}{\pi^2}(\partial_0\phi)^3 + \frac{3}{4\pi^2}(\partial_0\phi)^4. \quad (39)$$

3.1. One-point function

The evaluation of the term of the effective Lagrangian proportional to $\partial\phi$ requires the computation of the diagram in Fig. 3. One has that

$$\mathcal{L}_1 = -i\text{Tr}[S\Gamma]\Big|_A, \quad (40)$$

where the subscript means that in the evaluation of the trace only the terms linear in A must be included. We find that

$$\mathcal{L}_1 = -i\oint d\ell \left[\tilde{V} \cdot A \frac{V \cdot \ell}{D^*} - V \cdot A \frac{\tilde{V} \cdot \ell}{D} \right], \quad (41)$$

where we have defined

$$\oint d\ell = \frac{2}{\pi} \sum_{N_c, N_f} \int \frac{d\mathbf{v}}{4\pi} \int_{-\delta}^{+\delta} \frac{d\ell_{\parallel}}{2\pi} (\mu + \ell_{\parallel})^2 \int_{-\infty}^{\infty} \frac{d\ell_0}{2\pi}, \quad (42)$$

where δ is a cutoff that we shall set equal to μ , see [3]. In Eq. (41) we have also used [29]

$$D = V \cdot \ell \tilde{V} \cdot \ell - \Delta^2 + i\epsilon, \quad (43)$$

and for future convenience we define the quantity

$$L_0 = (2\mu + \tilde{V} \cdot \ell)(-2\mu + V \cdot \ell) - \Delta^2 - i\epsilon. \quad (44)$$

The sum over flavor and color degrees of freedom is straightforward and we obtain

$$\begin{aligned} \mathcal{L}_1 = & -\frac{2i}{\pi} \int \frac{d\mathbf{v}}{4\pi} \int \frac{d^2\ell}{(2\pi)^2} (\mu + \ell_{\parallel})^2 \left[\tilde{V} \cdot A \left(8 \frac{V \cdot \ell}{D(\Delta)^*} + \frac{V \cdot \ell}{D(-2\Delta)^*} \right) \right. \\ & \left. - V \cdot A \left(8 \frac{\tilde{V} \cdot \ell}{D(\Delta)} + \frac{\tilde{V} \cdot \ell}{D(-2\Delta)} \right) \right]. \end{aligned} \quad (45)$$

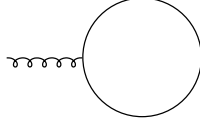


Figure 3. One loop diagram that contributes to the coefficient of the $\partial_0\phi$ term in the effective Lagrangian \mathcal{L}_1 in Eq. (36). The full line corresponds to the quark field. The wavy line corresponds to the external gauge field.

Employing the expressions reported in the Appendix B one can do the integration over the residual momentum and on the Fermi velocity. In this way one finds that

$$\mathcal{L}_1 = \left(-\frac{3}{\pi^2}\mu^3 + \frac{6}{\pi^2}\mu^2\Delta \right) \partial_0\phi. \quad (46)$$

Notice that at the leading order in μ the expression above for the term proportional to $\partial_0\phi$ agrees with the corresponding expression reported in Eq. (39).

3.2. Two-point function

The Lagrangian involving two gauge fields is given by

$$\mathcal{L}_2 = -i \left(\text{Tr}[S\Gamma] - \frac{1}{2}\text{Tr}[S\Gamma S\Gamma] \right) \Big|_{A^2}, \quad (47)$$

where in the evaluation of the trace one has to consider only terms quadratic in A . The corresponding diagrams are reported in Fig. 4. The diagram in Fig. 4a) gives

$$\text{Tr}[S\Gamma] \Big|_{A^2} = \oint d\ell P^{\mu\nu} A_\mu A_\nu \left[\frac{\Delta^2 + \tilde{V} \cdot \ell (V \cdot \ell - 2\mu)}{L_0 D} + (V \rightarrow \tilde{V}) \right], \quad (48)$$

where L_0 has been defined in Eq. (44) and $(V \rightarrow \tilde{V})$ actually means an expression that is obtained by replacing $(V \rightarrow \tilde{V}, \ell_\parallel \rightarrow -\ell_\parallel, \epsilon \rightarrow -\epsilon)$. Hereafter we shall always use this way of writing in order to simplify the notation.

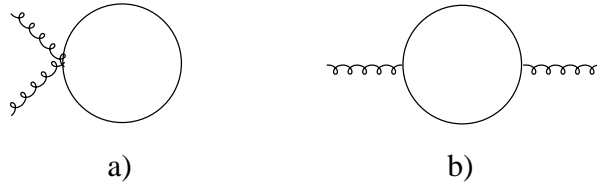


Figure 4. One loop diagrams that contribute to the coefficients of the A_0^2 term, and of the \mathbf{A}^2 term of the effective Lagrangian \mathcal{L}_2 in Eq. (36). Full lines are fermionic fields. Wavy lines are gauge fields.

The diagram in Fig. 4b) gives

$$\text{Tr}[S\Gamma S\Gamma] \Big|_{A^2} = \oint d\ell \left[\left(\frac{(\tilde{V} \cdot A)^2 (l_0 + \ell_\parallel)^2}{D^2} - \frac{(\tilde{V} \cdot A V \cdot A) \Delta^2}{D^2} \right) + (V \rightarrow \tilde{V}) \right], \quad (49)$$

and evaluating the integrals using the expressions reported in the Appendix B we have that

$$\begin{aligned}\mathcal{L}_2 &\simeq \frac{9\mu^2}{2\pi^2} \left(1 - 2\frac{\Delta^2}{\mu^2}\right) A_0^2 - \frac{3\mu^2}{2\pi^2} \left(1 - 2.1\frac{\Delta^2}{\mu^2}\right) \mathbf{A}^2 + \mathcal{O}\left(\frac{\Delta^2}{\mu^2} \log(\Delta/\mu)\right) \\ &= \frac{1}{2}m_D^2 A_0^2 - \frac{1}{2}m_M^2 \mathbf{A}^2.\end{aligned}\quad (50)$$

Then the speed of the NG boson is given by $c_s = m_M/m_D$ and the corresponding plot is reported in Fig. 5. For vanishing Δ we have that $c_s = 1/\sqrt{3}$ meaning that the system is scale invariant. The effect of a nonvanishing Δ is to increase the speed of the NG boson.

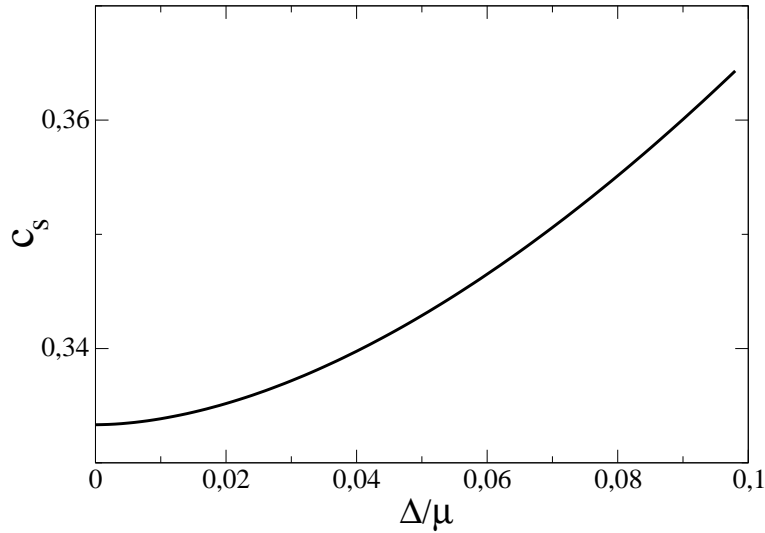


Figure 5. Speed of the NG boson associated to the breaking of $U(1)_B$ as a function of Δ/μ .

3.3. Three-point function

The Lagrangian describing the interaction of three NG bosons is formally given by

$$\mathcal{L}_3 = -i \left(\text{Tr}[S\Gamma] - \frac{1}{2}\text{Tr}[STST] + \frac{1}{3}\text{Tr}[STSTST] \right) \Big|_{A^3}, \quad (51)$$

and corresponds to the evaluation of the diagrams in Fig. 6. The diagrams 6a) and 6b) contribute exclusively to the term $A_0 \mathbf{A}^2$, while the diagram 6c) gives the term proportional to A_0^3 . The contribution of the diagram in Fig. 6a) is given by

$$\begin{aligned}\text{Tr}[S\Gamma] \Big|_{A^3} &= \oint d\ell P^{\mu\nu} A_\mu A_\nu V \cdot A \left[\frac{L_0 \tilde{V} \cdot \ell + (V \cdot \ell - \tilde{V} \cdot \ell - 4\mu)(D + 2\Delta^2 - 2\mu \tilde{V} \cdot \ell)}{DL_0^2} \right. \\ &\quad \left. - (V \rightarrow \tilde{V}) \right].\end{aligned}\quad (52)$$

The contribution of the diagram in Fig. 6b) is given by

$$\text{Tr}[STST] \Big|_{A^3} = -2 \oint d\ell P^{\mu\nu} A_\mu A_\nu V \cdot A \left[\frac{2\Delta^2 \tilde{V} \cdot \ell + (V \cdot \ell - 2\mu)((\tilde{V} \cdot \ell)^2 - \Delta^2)}{D^2 L_0} \right]$$

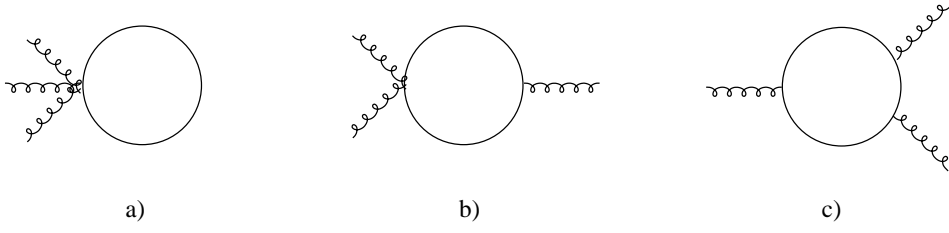


Figure 6. One loop diagrams that contribute to the effective Lagrangian \mathcal{L}_3 in Eq. (36). Notice that the only contribution to A_0^3 comes from diagram c). Full lines are fermionic fields. Wavy lines are gauge fields.

$$- (V \rightarrow \tilde{V}) \Big] . \quad (53)$$

The contribution of the diagram in Fig. 6c) is given by

$$\begin{aligned} \text{Tr}[S\Gamma S\Gamma S\Gamma] \Big|_{A^3} = & \int d\ell \left[\frac{-(V \cdot A)^3 (\tilde{V} \cdot \ell)^3 - \Delta^2 (\tilde{V} \cdot A)^2 V \cdot A V \cdot \ell}{D^3} \right. \\ & \left. + \frac{2\Delta^2 (\tilde{V} \cdot A)^2 (V \cdot A) (\tilde{V} \cdot \ell)}{D^3} - (V \rightarrow \tilde{V}) \right] . \end{aligned} \quad (54)$$

Evaluating the integrals with the help of the expressions reported in the Appendix B one finds that

$$\mathcal{L}_3 = \frac{3\mu}{\pi^2} \left(1 - \frac{\Delta^2}{\mu^2} \right) A_0 \mathbf{A}^2 - \frac{3\mu}{\pi^2} \left(1 - \frac{3\Delta^2}{2\mu^2} \right) A_0^3 . \quad (55)$$

3.4. Four-point function

The Lagrangian describing the interaction of four NG bosons is formally given by

$$\mathcal{L}_4 = -i \left(\text{Tr}[S\Gamma] - \frac{1}{2} \text{Tr}[S\Gamma S\Gamma] + \frac{1}{3} \text{Tr}[S\Gamma S\Gamma S\Gamma] + \frac{1}{4} \text{Tr}[S\Gamma S\Gamma S\Gamma S\Gamma] \right) \Big|_{A^4} , \quad (56)$$

and corresponds to the sum of the diagrams reported in Fig. 7. All these diagrams, with the exception of the diagram 7e), originate from the non-local vertices Γ_2 , Γ_3 and Γ_4 , and therefore can give contributions to the coefficients of the terms with $A_0^2 \mathbf{A}^2$ and \mathbf{A}^4 . The diagram in Fig. 7e) contributes to the coefficient of the term proportional to A_0^4 .

The contribution of the diagram in Fig. 7a) is given by

$$\begin{aligned} \text{Tr}[S\Gamma] \Big|_{A^4} = & \int d\ell P^{\mu\nu} A_\mu A_\nu (V \cdot A) \left\{ (V \cdot A) \left[\frac{-\tilde{V} \cdot \ell (\tilde{V} \cdot \ell + 2\mu)}{DL_0^2} \right. \right. \\ & \left. \left. + Z \frac{(V \cdot \ell - 2\mu)^2 + (\tilde{V} \cdot \ell + 2\mu)^2}{DL_0^3} \right] \right. \\ & \left. + \tilde{V} \cdot A \left[\frac{2Z - \Delta^2}{DL_0^2} - 2Z \frac{(V \cdot \ell - 2\mu)(\tilde{V} \cdot \ell + 2\mu)}{DL_0^3} \right] + (V \rightarrow \tilde{V}) \right\} , \end{aligned} \quad (57)$$

where $Z = (D + 2\Delta^2 - 2\mu \tilde{V} \cdot \ell)$.

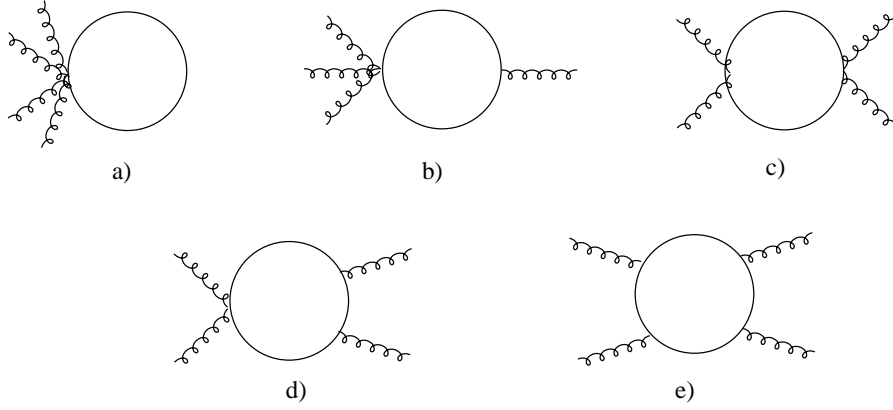


Figure 7. One loop diagrams that contribute to the effective Lagrangian \mathcal{L}_4 in Eq. (36). Full lines are fermionic fields. Wavy lines are gauge fields.

The diagram in Fig.7b) gives

$$\begin{aligned} & \oint d\ell P^{\mu\nu} A_\mu A_\nu \left\{ (V \cdot A)^2 \left[\frac{2\Delta^2 (V \cdot \ell)^2 + 3(\tilde{V} \cdot \ell)^2 - 3\mu V \cdot \ell + 5\mu \tilde{V} \cdot \ell + 4\mu^2}{D^2 L_0^2} \right] \right. \\ & + 2\tilde{V} \cdot AV \cdot A \left[\frac{-(D + 2\Delta^2)^2 + \mu(V \cdot \ell(\tilde{V} \cdot \ell)^2 - \Delta^2(V \cdot \ell - 3\tilde{V} \cdot \ell)) - 4(\tilde{V} \cdot \ell)^2 \mu^2}{D^2 L_0^2} \right] \\ & \left. + (V \rightarrow \tilde{V}) \right\}, \end{aligned} \quad (58)$$

while the diagram in Fig.7c) gives

$$\begin{aligned} & \oint d\ell P^{\mu\nu} A_\mu A_\nu P^{\alpha\beta} A_\alpha A_\beta \left[\frac{D + 2\Delta^2 + 4\mu^2(\tilde{V} \cdot \ell)^2 - 4\Delta^2 \mu(2\tilde{V} \cdot \ell + \mu)}{D^2 L_0^2} \right. \\ & \left. + \frac{V \cdot \ell(6\Delta^2 \tilde{V} \cdot \ell - 4\mu(\tilde{V} \cdot \ell)^2 + 4\Delta^2 \mu)}{D^2 L_0^2} + (V \rightarrow \tilde{V}) \right]. \end{aligned} \quad (59)$$

Both these diagrams are determined by evaluating

$$\text{Tr}[STST] \Big|_{A^4}. \quad (60)$$

The diagram in Fig.7d) gives

$$\begin{aligned} \text{Tr}[STSTST] \Big|_{A^4} &= \oint d\ell P^{\mu\nu} A_\mu A_\nu \left\{ (V \cdot A)^2 \left[\frac{3(\tilde{V} \cdot \ell)^3(V \cdot \ell - 2\mu)}{D^3 L_0} \right] \right. \\ &+ \Delta^2 \frac{2(V \cdot \ell)^2 + 7(\tilde{V} \cdot \ell)^2 - 2\mu V \cdot \ell + 4\mu \tilde{V} \cdot \ell}{D^3 L_0} \\ &- \tilde{V} \cdot AV \cdot A \left[\frac{9D + 12\Delta^2 + 4\mu(V \cdot \ell - 2\tilde{V} \cdot \ell)}{DL_0^3} \right] \\ &\left. + (V \rightarrow \tilde{V}) \right\}. \end{aligned} \quad (61)$$

Finally, the diagram in Fig.7e) gives

$$\begin{aligned} \text{Tr}[STSTSTST] \Big|_{A^4} &= \oint d\ell \frac{1}{D^4} \left[(V \cdot A)^4 (\tilde{V} \cdot \ell)^4 - 3(V \cdot A)^3 \tilde{V} \cdot A \Delta^2 (\tilde{V} \cdot \ell)^2 \right. \\ &- V \cdot A (\tilde{V} \cdot A)^3 \Delta^2 (V \cdot \ell)^2 + (V \cdot A)^2 (\tilde{V} \cdot A)^2 (2D + 3\Delta^2) \\ &\left. + (V \rightarrow \tilde{V}) \right]. \end{aligned} \quad (62)$$

After a long but straightforward calculation we find that at the leading order in μ

$$\mathcal{L}_4 = \frac{3}{4\pi^2} A_0^4 + \frac{3}{4\pi^2} \mathbf{A}^4 - \frac{3}{2\pi^2} A_0^2 \mathbf{A}^2. \quad (63)$$

In this case we restricted our calculation, for simplicity, to evaluate the leading terms in μ .

4. Higgs field and interaction terms

The fluctuations in the absolute value of the condensate are described by the Higgs field ρ defined in Eq. (10). The effective Lagrangian for this field, including the interaction terms with the NG bosons, can be determined by means of the same strategy employed in the previous section.

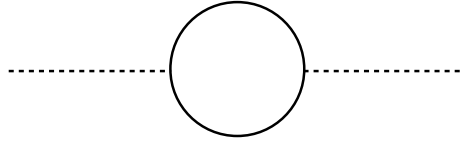


Figure 8. Diagrams that contribute to the one loop self-energy of the Higgs field. Full lines are fermionic fields. Dashed lines correspond to the Higgs field, ρ .

From the Γ expansion in Eq. (33) we obtain the self-energy diagram in Fig. 8 which gives for vanishing external momentum of the Higgs field

$$\mathcal{L}_{\rho\rho} = -\frac{6}{\pi^2} \mu^2 \rho(x)^2, \quad (64)$$

which is the mass term for the ρ field. However, the actual mass of the Higgs is not proportional to the chemical potential: as we shall show below one needs a wave function renormalization in order to put the Lagrangian in the canonical form. We notice that the ρ field does not carry color and flavor indices. In order to obtain Eq. (64) (and all the expressions below), color and flavor indices have been properly contracted and the resulting Lagrangian is expressed in terms of colorless fields.

Considering the fluctuation of the chemical potential given by $\partial_0\phi$, the Lagrangian above turns into

$$\mathcal{L}_{\rho\rho} = -\frac{6}{\pi^2} \tilde{\mu}^2 \rho(x)^2 = -\frac{6}{\pi^2} (\mu - \partial_0\phi)^2 \rho(x)^2, \quad (65)$$

which automatically gives part of the interaction of the Higgs field with the NG bosons. The remaining interaction terms of two Higgs field with two NG bosons correspond to a coupling $(\partial_i\phi)^2 \rho(x)^2$. These interaction terms, as well as the interaction terms in Eq. (65), can be obtained from the diagrams in Fig. 9.

Evaluating these diagrams one obtains that

$$\mathcal{L}_{\rho\rho\phi\phi} = -\frac{6}{\pi^2} \left[(\mu - \partial_0\phi)^2 - \frac{1}{3} (\partial_i\phi)^2 \right] \rho(x)^2, \quad (66)$$

which describes the interaction terms among the Higgs fields and two NG bosons at the leading order in μ . Notice that the term $\rho \partial_\mu \phi \partial^\mu \phi$ describing the interaction between

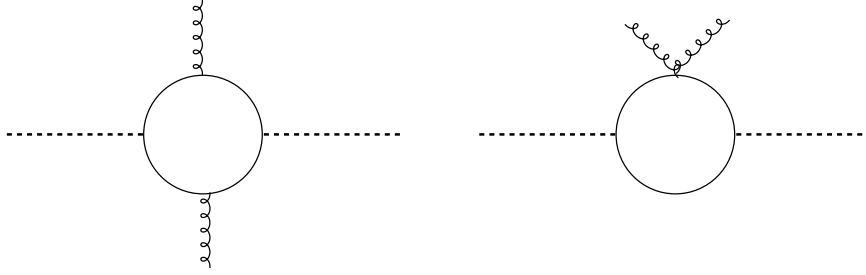


Figure 9. Interaction of the Higgs field with two NG bosons at the one loop level. Full lines correspond to fermionic fields, dashed lines correspond to the Higgs field and wavy lines correspond to the gauge field $A^\mu = \partial^\mu \phi$.

one Higgs field and two NG bosons is missing. This is basically due to the fact that one cannot have a term like $\mu^2 \rho$ from the loop expansion. The fact that in our case terms linear in the ρ field are missing, leads to an interesting effect. Let us consider a Lagrangian with terms up to quadratic order in the NG bosons and Higgs fields. Since the Lagrangian is quadratic in the massive Higgs field one can integrate them out from the theory obtaining the actual low energy Lagrangian of the system. In the CFL phase this does not lead to a modification of the effective Lagrangian of the NG bosons. In particular the velocity of the NG bosons at the leading order in μ is $c_s = \sqrt{\frac{1}{3}}$. This is rather different from what happens in the non-relativistic case [24], where it was shown that integrating out the Higgs mode one obtains the modification of the speed of sound first determined in [30].

In order to clarify this point, we compare our Lagrangian with the quadratic non-relativistic Lagrangian. Schematically the result of [24] can be written as

$$\mathcal{L}_{N.R.}(\rho, Y) = \frac{A}{2} \rho^2 + B \rho Y + \frac{C}{2} Y^2 + D Y, \quad (67)$$

where A, B, C, D are some coefficients and $Y = \partial_0 \phi + \frac{(\nabla \phi)^2}{2m}$, with m the mass of the non-relativistic fermions. The speed of the NG bosons is given by

$$c_s^2 = -\frac{D}{mC}. \quad (68)$$

Integrating out the ρ field, the effective Lagrangian for the ϕ field turns out to be

$$\mathcal{L}_{N.R.}(\phi) = \frac{B^2 + AC}{2A} Y^2 + D Y, \quad (69)$$

and the speed of sound is modified to

$$c_s^2 = -\frac{DA}{m(B^2 + AC)}. \quad (70)$$

Notice that if the coupling B vanishes, then the speed of the NG bosons remains the same one has in Eq. (68).

In our case the LO Lagrangian quadratic in the ρ field is given by Eq. (66) and there is no term that couples one ρ field with two NG bosons, *i.e.* expanding the Lagrangian one finds that the analogous of the coefficient B in Eq. (67) is missing. Therefore, integrating out the ρ field does not change the effective Lagrangian for the NG bosons.

From the diagrams in Fig. 9 we can determine the kinetic terms of the effective Lagrangian of the Higgs field. Considering soft momenta of the Higgs field, $p \ll \Delta$, and expanding up to the order $(p/\Delta)^2$, we obtain

$$\mathcal{L}_K(\rho) = \frac{1}{2} \frac{3\mu^2}{4\pi^2} \frac{1}{\Delta^2} \left[(\partial_0 \rho)^2 - \frac{1}{3} (\partial_i \rho)^2 \right] - \frac{1}{2} \frac{12\mu^2}{\pi^2} \rho^2. \quad (71)$$

By a wave function renormalization we can cast the above expression into canonical form and we obtain that the mass of the Higgs field is given by

$$m_\rho = 4\Delta, \quad (72)$$

which means that the mass of the Higgs is twice the fermionic excitation energy. This is analogous to the result obtained in the chiral sector, where the masses of the mesons turn to be equal to twice the effective mass of the quarks, see [20].

From the expression of the mass of the Higgs, it seems unlikely that an NG boson could excite the ρ mode. The reason is that the scale of the field ϕ is T and in compact stars $T \ll \Delta$. Even if thermal NG bosons in general cannot excite the Higgs, for vortex-NG boson interaction one has to properly take into account the fact that as one moves inside a vortex, the actual value of the mass of the Higgs field should decrease. The reason is that as one moves inside a vortex, the value of the condensate becomes smaller and smaller and the mass of the ρ field should decrease accordingly. Therefore at a certain point it should happen that $m_\rho < T$ and it may become possible for an NG boson to excite the ρ mode. However, a vortex is a modulation of the ρ field itself, and therefore the discussion of the interaction between NG bosons and Higgs field in a vortex is subtle. We postpone to future work the analysis of this situation.

We now try to give a general expression of the interaction terms between two NG bosons and the Higgs fields. Let us first neglect space variations of the ϕ field. Then, expanding the effective action one has terms like

$$\mathcal{L}(\rho, \partial_0 \phi) = \sum_{n \geq 2} c_n \frac{(\mu - \partial_0 \phi)^2 \rho^n}{\Delta^{n-2}}, \quad (73)$$

where c_n are some dimensionless coefficients. This expression is simply due to the fact that any term in the effective Lagrangian is multiplied by $\tilde{\mu}^2$, which comes from the phase space integration, while the denominator comes from dimensional analysis.

Now we want to derive the expression of the terms with space derivatives of the NG bosons. We know from Eq. (66), that for $n = 2$, one has to replace $(\mu - \partial_0 \phi)^2$ with $(\mu - \partial_0 \phi)^2 - 1/3(\nabla \phi)^2$, where the coefficient $1/3$ is precisely the square of the speed of sound. In other words the correct metric for the propagation of NG bosons is the *acoustic metric*, $g_{\mu\nu} = \text{diag}(1, -1/3, -1/3, -1/3)$. Then, we introduce the four vector $X^\mu = (\mu - \partial_0 \phi, \nabla \phi)$ and our guess is that the Lagrangian at the leading order in μ is given by

$$\mathcal{L}(\rho, X^\mu) = \sum_{n \geq 2} c_n \frac{X^\mu X^\nu g_{\mu\nu} \rho^n}{\Delta^{n-2}}. \quad (74)$$

The reasoning is that any perturbation propagating in the medium feels the presence of the background which induces the acoustic metric $g_{\mu\nu}$ [31, 32]. We shall investigate

in more detail this guess in future work, however it was already shown in [32] that the free Lagrangian of NG bosons in the CFL phase can be written employing the acoustic metric. Therefore Eq. (74) seems to be an educated guess.

If the expression above is correct it allows to readily determine the interaction between any number of Higgs fields with two NG bosons in a straightforward way. As an example we can determine the interaction between three Higgs field and two NG bosons. We first evaluate the diagram in Fig. 10, which gives the interaction among three Higgs field

$$\mathcal{L}_{\rho\rho\rho} = -\frac{2\mu^2}{\pi^2\Delta}\rho^3, \quad (75)$$

then, upon replacing $\mu^2 \rightarrow X^\mu X^\nu g_{\mu\nu}$ we obtain the LO interaction Lagrangian

$$\mathcal{L}_{\rho\rho\rho\phi\phi} = -\frac{2X^\mu X^\nu g_{\mu\nu}}{\pi^2\Delta}\rho^3. \quad (76)$$

It is not clear whether this reasoning can be extended in order to include subleading corrections of the order Δ/μ . In principle one would expect that the leading effect should be to perturb the metric $g_{\mu\nu}$, but further investigation in this direction is needed.

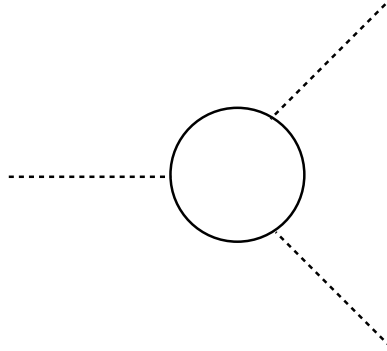


Figure 10. Interaction of three Higgs fields. Full lines correspond to fermionic fields, dashed lines correspond to the Higgs fields.

5. Conclusions

The low energy properties of cold and dense color flavor locked quark matter are determined by the NG bosons associated with the breaking of the $U(1)_B$ symmetry. At the very low temperatures expected in compact stars, the NG bosons probably give the leading contribution to the transport properties of the system. Therefore the detailed knowledge of their effective Lagrangian is important to precisely determine the transport coefficients. We have determined the effective Lagrangian for this field starting from a microscopic theory. As high energy theory we have considered the Nambu-Jona-Lasinio model with a local four-Fermi interaction with the quantum numbers of one gluon exchange. Then we have gauged the $U(1)_B$ symmetry introducing a fictitious gauge field. Finally we have integrated out the fermionic degrees of freedom by means of the HDET.

We have confirmed the results of Ref. [23], which were based on symmetry arguments, and extended including next to leading terms of order $(\Delta/\mu)^2$. These corrections are relevant because are related with the breaking of the conformal symmetry of the system and can be significant large for matter at non-asymptotic densities. For this reason, the present work paves the way for a more detailed calculation of the transport properties of CFL quark matter, including the scale breaking effects.

We have also determined the interaction of the NG bosons with the Higgs mode, *i.e.* with the collective mode associated with fluctuations of $|\Delta|$. These interactions are relevant in the calculation of the interaction of the NG bosons with vortices. Indeed, vortices can be described as space modulation of $|\Delta|$. A preliminary study of the mutual friction force in the CFL phase has been done in Ref. [33]. However, in that calculation of the mutual friction force only the elastic scattering of NG bosons on vortices has been taken into account. Since we have determined the full low energy Lagrangian, we are now in a position to evaluate in more detail the interaction of NG bosons with vortices, including non-elastic scattering, however we postpone the analysis of vortex-phonon interaction to future work. Indeed the treatment of this interaction is non-trivial, moreover superfluid vortices which wind the $U(1)_B$ are topologically stable but according to the result of the Ginzburg-Landau analysis of Ref. [34] they are dynamically unstable.

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Appendix A. Interaction vertices

In the HDET fermions interact with gauge field by the following vertices. The minimal coupling is due to the vertex

$$\Gamma_1 = \begin{pmatrix} -V \cdot A & 0 \\ 0 & \tilde{V} \cdot A \end{pmatrix}. \quad (\text{A.1})$$

Non minimal couplings are due to the expansion of the non-local interaction in Eq. (21). Expanding in the number of gauge fields one has the term with two gauge fields:

$$\Gamma_2 = \frac{P^{\mu\nu} A_\mu A_\nu}{L_0} \begin{pmatrix} -2\mu + V \cdot \ell & -\Delta \\ -\Delta & 2\mu + \tilde{V} \cdot \ell \end{pmatrix}, \quad (\text{A.2})$$

three gauge fields:

$$\Gamma_3 = -\frac{\Gamma_2}{L_0}[V \cdot A(2\mu + \tilde{V} \cdot \ell) + \tilde{V} \cdot A(2\mu - V \cdot \ell)] - \frac{\Gamma_1}{L_0}P^{\mu\nu}A_\mu A_\nu, \quad (\text{A.3})$$

and finally the interaction with four gauge fields:

$$\Gamma_4 = \frac{\Gamma_3}{L_0}[V \cdot A(2\mu + \tilde{V} \cdot \ell) + \tilde{V} \cdot A(2\mu - V \cdot \ell)]. \quad (\text{A.4})$$

In the present article we do not consider interactions with more than four gauge fields.

Appendix B. Integrals

In this appendix we evaluate some integrals used in the evaluation of the effective Lagrangians for the NG boson and for the Higgs field. For the integrals in ℓ_0 and ℓ_\parallel we define

$$\int d^2\ell \equiv \int_{-\mu}^{+\mu} d\ell_\parallel \int_{-\infty}^{+\infty} d\ell_0. \quad (\text{B.1})$$

We have that

$$\int d^2\ell \frac{1}{D(\ell)^{n+1}} = (-1)^{n+1} \frac{i\pi}{n\Delta^{2n}} \quad \text{for } n \geq 1 \quad (\text{B.2})$$

$$\int d^2\ell \frac{(V \cdot \ell)^2}{D(\ell)^n} = \int d^2\ell \frac{(\tilde{V} \cdot \ell)^2}{D(\ell)^n} = -i\pi \delta_{n1}, \quad (\text{B.3})$$

where $D(\ell) = V \cdot \ell \tilde{V} \cdot \ell - \Delta^2 + i\epsilon$. We also need the integrals

$$\int d^2\ell \frac{1}{L_0(\ell)} = -i\pi \log \left(\frac{\mu + \sqrt{\mu^2 + \Delta^2}}{3\mu + \sqrt{9\mu^2 + \Delta^2}} \right), \quad (\text{B.4})$$

$$\int d^2\ell \frac{1}{L_0(\ell)^2} = i\pi \frac{\mu}{2\Delta^2} \left(\frac{1}{\sqrt{\mu^2 + \Delta^2}} - \frac{3}{\sqrt{9\mu^2 + \Delta^2}} \right), \quad (\text{B.5})$$

$$\int d^2\ell \frac{1}{L_0(\ell)^3} = -i\pi \frac{\mu}{8\Delta^2} \left(\frac{2\mu^2 + 3\Delta^2}{(\mu^2 + \Delta^2)^{3/2}} - \frac{9(6\mu^2 + \Delta^2)}{(9\mu^2 + \Delta^2)^{3/2}} \right), \quad (\text{B.6})$$

where

$$L_0 = (2\mu + \tilde{V} \cdot \ell)(-2\mu + V \cdot \ell) - \Delta^2 - i\epsilon. \quad (\text{B.7})$$

Appendix B.1. Angular integrals

Considering a general vector A^μ , for the integrals involving terms of order A we have that

$$H_0 = \int \frac{d\mathbf{V}}{4\pi} V \cdot A = A_0. \quad (\text{B.8})$$

At the order A^2 we have

$$H_1 = \int \frac{d\mathbf{v}}{4\pi} P^{\mu\nu} A_\mu A_\nu = -\frac{2}{3} \mathbf{A}^2, \quad (\text{B.9})$$

$$H_2 = \int \frac{d\mathbf{v}}{4\pi} (V \cdot A)^2 = A_0^2 + \frac{1}{3} \mathbf{A}^2, \quad (\text{B.10})$$

$$H_3 = \int \frac{d\mathbf{v}}{4\pi} V \cdot A \tilde{V} \cdot A = A_0^2 - \frac{1}{3} \mathbf{A}^2. \quad (\text{B.11})$$

At the order A^3 we have

$$H_4 = \int \frac{d\mathbf{v}}{4\pi} P^{\mu\nu} A_\mu A_\nu V \cdot A = -\frac{2}{3} A_0 \mathbf{A}^2, \quad (\text{B.12})$$

$$H_5 = \int \frac{d\mathbf{v}}{4\pi} (V \cdot A)^3 = A_0^3 + A_0 \mathbf{A}^2, \quad (\text{B.13})$$

$$H_6 = \int \frac{d\mathbf{v}}{4\pi} (V \cdot A)(\tilde{V} \cdot A)^2 = A_0^3 - \frac{1}{3} A_0 \mathbf{A}^2. \quad (\text{B.14})$$

At the order A^4 we have

$$H_7 = \int \frac{d\mathbf{v}}{4\pi} (V \cdot A)^4 = A_0^4 + 2A_0^2 \mathbf{A}^2 + \frac{1}{5} \mathbf{A}^4, \quad (\text{B.15})$$

$$H_8 = \int \frac{d\mathbf{v}}{4\pi} (V \cdot A)(\tilde{V} \cdot A)^3 = A_0^4 - \frac{1}{5} A_0^2 \mathbf{A}^2, \quad (\text{B.16})$$

$$H_9 = \int \frac{d\mathbf{v}}{4\pi} (V \cdot A)^2 (\tilde{V} \cdot A)^2 = A_0^4 - \frac{2}{3} A_0^2 \mathbf{A}^2 + \frac{1}{5} \mathbf{A}^4, \quad (\text{B.17})$$

$$H_{10} = \int \frac{d\mathbf{v}}{4\pi} P^{\mu\nu} A_\mu A_\nu (\tilde{V} \cdot A)^2 = -\frac{2}{3} A_0^2 \mathbf{A}^2 - \frac{2}{15} \mathbf{A}^4, \quad (\text{B.18})$$

$$H_{11} = \int \frac{d\mathbf{v}}{4\pi} P^{\mu\nu} A_\mu A_\nu P^{\rho\sigma} A_\rho A_\sigma = \frac{8}{15} \mathbf{A}^4, \quad (\text{B.19})$$

$$H_{12} = \int \frac{d\mathbf{v}}{4\pi} P^{\mu\nu} A_\mu A_\nu \tilde{V} \cdot A V \cdot A = -\frac{2}{3} A_0^2 \mathbf{A}^2 + \frac{2}{15} \mathbf{A}^4. \quad (\text{B.20})$$

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